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EXPLICIT DETERMINATION OF LATERAL-DIRECTIONAL STABILITY AND CONTROL DERIVATIVES BY SIMULTANEOUS TIME VECTOR ANALYSIS OF TWO MANEUVERS

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16. Abstract

An extension of the time vector technique for determining stability and control derivatives from flight data is formulated. The technique provides for explicit determination of derivatives by means of simultaneous analysis of two maneuvers which differ by a dependent control input. The control derivatives for the dependent input are also explicitly determined.

This extended technique is preferable to the application of the time vector method to single maneuvers in that no estimates of derivatives are required. An example illustrating the application of the technique is given.

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INTRODUCTION

The time vector method has been applied to stability and control analysis for more than 20 years. As initially developed, the time vector method used a set of known or estimated derivatives to investigate aircraft stability (ref. 1). Shortly thereafter, the time vector method was applied to the analysis of flight data to determine stability derivatives (ref. 2). In past applications of the time vector method, only two unknowns in any single equation of motion could be solved for, and therefore only stability derivatives of the longitudinal short-period mode could be solved for explicitly. Analysis of the lateral-directional mode required that the values of some derivatives be assumed in order to solve for the remaining derivatives.

This report extends the time vector method to the simultaneous analysis of two maneuvers which differ by some dependent control movement, such as stability augmentation system feedback. As a result of the simultaneous analysis of the lateral-directional mode, stability derivatives and dependent control derivatives can be determined explicitly. In the linear case, no estimates of any derivatives are required.

An example of the application of the technique to flight data is included.

SYMBOLS

Physical quantities in this report are given in the International System of Units (SI) and parenthetically in U.S. Customary Units. The measurements were taken in Customary Units. Factors relating the two systems are presented in reference 3.

a_t	transverse acceleration at the center of gravity, g
g	acceleration due to gravity, m/sec^2 (ft/sec^2)
I_X, I_Z	moments of inertia about X- and Z-body axes, respectively, $kg\text{-}m^2$ ($slug\text{-}ft^2$)
I_{XZ}	product of inertia referred to the body X- and Z-axes, $kg\text{-}m^2$ ($slug\text{-}ft^2$)

$L = \dfrac{\text{Rolling moment}}{I_X}$, rad/sec^2

$L_{p_o}, L_{p_{L_r}}, L_{p_{L_\delta}}$ constants of L_p equation

$L_{\beta_o}, L_{\beta_{L_r}}, L_{\beta_{L_\delta}}$ constants of L_β equation

$N = \dfrac{\text{Yawing moment}}{I_Z}$, rad/sec^2

$N_{p_o}, N_{p_{N_r}}, N_{p_{N_\delta}}$ constants of N_p equation

$N_{\beta_o}, N_{\beta_{N_r}}, N_{\beta_{N_\delta}}$ constants of N_β equation

p, r roll rate and yaw rate, respectively, rad/sec

V velocity, m/sec (ft/sec)

$Y = \dfrac{\text{Side force}}{(\text{Aircraft mass})(V)}$, 1/sec

$Y_{p_o}, Y_{p_{Y_r}}, Y_{p_{Y_\delta}}$ constants of Y_p equation

$Y_{\beta_o}, Y_{\beta_{Y_r}}, Y_{\beta_{Y_\delta}}$ constants of Y_β equation

α_o initial condition angle of attack at the center of gravity, rad

β angle of sideslip at the center of gravity, rad

Δ determinant

δ control deflection

δ_a aileron deflection, aileron deflection that produces right roll is positive, rad

$\Phi_{i/j}, \left|\dfrac{i}{j}\right|$ phase angle and amplitude ratio of quantity i relative to quantity j

φ bank angle, rad

Subscripts:

p, r, β, δ — partial derivatives with respect to subscripted variables

A dot over a quantity denotes the time derivative of that quantity.

MANEUVER REQUIREMENTS

Two oscillatory maneuvers which are free of pilot-induced inputs (after initiation of the maneuver) and with damping ratios of less than approximately 0.3 are required. The maneuvers must differ by some dependent control variable, for example, roll stability augmentation system (SAS) on and roll SAS off. Another possible combination would be two maneuvers with the roll SAS on with different feedback gains.

The dependent control requirement is easily met, since most research and prototype aircraft have independently selectable roll and yaw SAS and some have pilot-variable SAS gains.

The oscillatory mode to be analyzed must be well separated from other mode shapes.

MATHEMATICAL DEVELOPMENT

The fundamental concepts and the relations that exist for the equations of motion in the time vector format are presented in reference 4.

Since a time vector solution of one maneuver can determine only two unknowns in each equation, past applications were explicit only for control-fixed longitudinal short-period maneuvers. Analysis of lateral-directional control-fixed maneuvers required the assumption of one derivative in each of the equations. However, two properly conditioned maneuvers can provide the solution of four unknowns, including a control derivative, in each equation without any use of estimates.

The development that follows assumes the general form of a dependent but different control input in each maneuver. The same equations apply when one of the maneuvers does not have a dependent control input.

In dimensional form, the linearized lateral-directional equations of motion are

$$\left.\begin{aligned}
\dot{p} - \frac{I_{XZ}}{I_X}\dot{r} &= L_\beta\beta + L_p p + L_r r + L_\delta\delta \\
\dot{r} - \frac{I_{XZ}}{I_Z}\dot{p} &= N_\beta\beta + N_p p + N_r r + N_\delta\delta \\
\dot{\beta} &= -r + \alpha_o p + \frac{g}{V}(\varphi + a_t) \\
\frac{g}{V}a_t &= Y_\beta\beta + Y_p p + Y_r r + Y_\delta\delta
\end{aligned}\right\} \qquad (1)$$

Analysis of the First Maneuver

The derivation that follows manipulates the time vector relations of the first maneuver to obtain the coefficients of two derivatives in terms of the two remaining derivatives. Throughout, the three equations of motion are treated similarly; therefore, only the $\dot{p}$ equation is used as an example.

Expressed in time vector format and with p arbitrarily selected as a base, the $\dot{p}$ equation becomes

$$\left|\frac{\dot{p}}{p}\right|\Phi_{\dot{p}/p} - \frac{I_{XZ}}{I_X}\left|\frac{\dot{r}}{p}\right|\Phi_{\dot{r}/p} = L_\beta\left|\frac{\beta}{p}\right|\Phi_{\beta/p} + L_p\left|\frac{p}{p}\right|\Phi_{p/p} + L_r\left|\frac{r}{p}\right|\Phi_{r/p} + L_\delta\left|\frac{\delta}{p}\right|\Phi_{\delta/p} \tag{2}$$

Since equation (2) represents a two-dimensional vector diagram, the equation can be written in cosine and sine components. In matrix notation this is

$$\begin{vmatrix} \left|\frac{\beta}{p}\right|\cos\Phi_{\beta/p} & \left|\frac{p}{p}\right|\cos\Phi_{p/p} \\ \left|\frac{\beta}{p}\right|\sin\Phi_{\beta/p} & \left|\frac{p}{p}\right|\sin\Phi_{p/p} \end{vmatrix}\begin{vmatrix} L_\beta \\ L_p \end{vmatrix} = \begin{vmatrix} \left|\frac{\dot{p}}{p}\right|\cos\Phi_{\dot{p}/p} - \frac{I_{XZ}}{I_X}\left|\frac{\dot{r}}{p}\right|\cos\Phi_{\dot{r}/p} - L_r\left|\frac{r}{p}\right|\cos\Phi_{r/p} - L_\delta\left|\frac{\delta}{p}\right|\cos\Phi_{\delta/p} \\ \left|\frac{\dot{p}}{p}\right|\sin\Phi_{\dot{p}/p} - \frac{I_{XZ}}{I_X}\left|\frac{\dot{r}}{p}\right|\sin\Phi_{\dot{r}/p} - L_r\left|\frac{r}{p}\right|\sin\Phi_{r/p} - L_\delta\left|\frac{\delta}{p}\right|\sin\Phi_{\delta/p} \end{vmatrix} \tag{3}$$

Solving equations (3) for the derivatives L_β and L_p as functions of L_r and L_δ results in the form

$$\left.\begin{aligned} L_\beta &= L_{\beta_o} + L_{\beta_{L_r}} L_r + L_{\beta_{L_\delta}} L_\delta \\ L_p &= L_{p_o} + L_{p_{L_r}} L_r + L_{p_{L_\delta}} L_\delta \end{aligned}\right\} \tag{4}$$

The constant coefficients of equations (4) to be determined from the first maneuver are

$$\left.\begin{aligned} L_{\beta_o} &= \frac{\left|\frac{p}{p}\right|\left[\left|\frac{\dot{p}}{p}\right|\sin\left(\Phi_{p/p} - \Phi_{\dot{p}/p}\right) - \frac{I_{XZ}}{I_X}\left|\frac{\dot{r}}{p}\right|\sin\left(\Phi_{p/p} - \Phi_{\dot{r}/p}\right)\right]}{\Delta} \\ L_{\beta_{L_r}} &= \frac{-\left|\frac{p}{p}\right|\left|\frac{r}{p}\right|\sin\left(\Phi_{p/p} - \Phi_{r/p}\right)}{\Delta} \\ L_{\beta_{L_\delta}} &= \frac{-\left|\frac{p}{p}\right|\left|\frac{\delta}{p}\right|\sin\left(\Phi_{p/p} - \Phi_{\delta/p}\right)}{\Delta} \\ L_{p_o} &= \frac{\left|\frac{\beta}{p}\right|\left[-\left|\frac{\dot{p}}{p}\right|\sin\left(\Phi_{\beta/p} - \Phi_{\dot{p}/p}\right) + \frac{I_{XZ}}{I_X}\left|\frac{\dot{r}}{p}\right|\sin\left(\Phi_{\beta/p} - \Phi_{\dot{r}/p}\right)\right]}{\Delta} \\ L_{p_{L_r}} &= \frac{\left|\frac{\beta}{p}\right|\left|\frac{r}{p}\right|\sin\left(\Phi_{\beta/p} - \Phi_{r/p}\right)}{\Delta} \\ L_{p_{L_\delta}} &= \frac{\left|\frac{\beta}{p}\right|\left|\frac{\delta}{p}\right|\sin\left(\Phi_{\beta/p} - \Phi_{\delta/p}\right)}{\Delta} \end{aligned}\right\} \tag{5}$$

where

$$\Delta = \left|\frac{\beta}{p}\right|\left|\frac{p}{p}\right| \sin\left(\Phi_{p/p} - \Phi_{\beta/p}\right)$$

The procedure for determining the derivative coefficients of the $\dot{r}$ and a_t equations is identical to the procedure for the $\dot{p}$ equation. The coefficients for the yawing moment and lateral-acceleration derivatives are then

$$\left.\begin{aligned}
N_{\beta_o} &= \frac{\left|\frac{p}{p}\right|\left[\left|\frac{\dot{r}}{p}\right| \sin\left(\Phi_{p/p} - \Phi_{\dot{r}/p}\right) - \frac{I_{XZ}}{I_Z}\left|\frac{\dot{p}}{p}\right| \sin\left(\Phi_{p/p} - \Phi_{\dot{p}/p}\right)\right]}{\Delta} \\
N_{\beta_{N_r}} &= L_{\beta_{L_r}} \\
N_{\beta_{N_\delta}} &= L_{\beta_{L_\delta}} \\
N_{p_o} &= \frac{\left|\frac{\beta}{p}\right|\left[-\left|\frac{\dot{r}}{p}\right| \sin\left(\Phi_{\beta/p} - \Phi_{\dot{r}/p}\right) + \frac{I_{XZ}}{I_Z}\left|\frac{\dot{p}}{p}\right| \sin\left(\Phi_{\beta/p} - \Phi_{\dot{p}/p}\right)\right]}{\Delta} \\
N_{p_{N_r}} &= L_{p_{L_r}} \\
N_{p_{N_\delta}} &= L_{p_{L_\delta}} \\
Y_{\beta_o} &= \frac{\frac{g}{V}\left|\frac{a_t}{p}\right|\left|\frac{p}{p}\right| \sin\left(\Phi_{p/p} - \Phi_{a_t/p}\right)}{\Delta} \\
Y_{\beta_{Y_r}} &= L_{\beta_{L_r}} \\
Y_{\beta_{Y_\delta}} &= L_{\beta_{L_\delta}} \\
Y_{p_o} &= \frac{-\frac{g}{V}\left|\frac{a_t}{p}\right|\left|\frac{\beta}{p}\right| \sin\left(\Phi_{\beta/p} - \Phi_{a_t/p}\right)}{\Delta} \\
Y_{p_{Y_r}} &= L_{p_{L_r}} \\
Y_{p_{Y_\delta}} &= L_{p_{L_\delta}}
\end{aligned}\right\} \qquad (6)$$

By using the time vector identities discussed in reference 4, only the time histories of the variables p, r, a_t, and δ are required.

Analysis of the Second Maneuver

Because of the dependent control input of the second maneuver, the aircraft response is different than in the first maneuver. However, substituting the results of the analysis of the first maneuver into the equations of motion leaves only two unknowns in each equation. The time vector analysis of the second maneuver thus provides for the explicit determination of four unknown derivatives in each equation.

Substituting the derivative expressions of equations (4) into the $\dot{p}$ equation (eq. (1)) results in the following:

$$\dot{p} - \frac{I_{XZ}}{I_X}\dot{r} = \underbrace{\left(L_{\beta_o} + L_{\beta_{L_r}} L_r + L_{\beta_{L_\delta}} L_\delta\right)}_{L_\beta} \beta + \underbrace{\left(L_{p_o} + L_{p_{L_r}} L_r + L_{p_{L_\delta}} L_\delta\right)}_{L_p} p + L_r r + L_\delta \delta \tag{7}$$

Expanding and rearranging,

$$\dot{p} - \frac{I_{XZ}}{I_X}\dot{r} = L_{\beta_o}\beta + L_{p_o}p + L_r\left(L_{\beta_{L_r}}\beta + L_{p_{L_r}}p + r\right) + L_\delta\left(L_{\beta_{L_\delta}}\beta + L_{p_{L_\delta}}p + \delta\right) \tag{8}$$

The $\dot{p}$ equation is now a function of only two unknowns, L_r and L_δ; therefore, a vector analysis yields an explicit solution for these derivatives.

Writing equation (8) in cosine and sine components and matrix notation results in

$$\begin{vmatrix} \left(L_{\beta_{L_r}}\left|\frac{\beta}{p}\right| \cos \Phi_{\beta/p} + L_{p_{L_r}}\left|\frac{p}{p}\right| \cos \Phi_{p/p} + \left|\frac{r}{p}\right| \cos \Phi_{r/p}\right) & \left(L_{\beta_{L_\delta}}\left|\frac{\beta}{p}\right| \cos \Phi_{\beta/p} + L_{p_{L_\delta}}\left|\frac{p}{p}\right| \cos \Phi_{p/p} + \left|\frac{\delta}{p}\right| \cos \Phi_{\delta/p}\right) \\ \left(L_{\beta_{L_r}}\left|\frac{\beta}{p}\right| \sin \Phi_{\beta/p} + L_{p_{L_r}}\left|\frac{p}{p}\right| \sin \Phi_{p/p} + \left|\frac{r}{p}\right| \sin \Phi_{r/p}\right) & \left(L_{\beta_{L_\delta}}\left|\frac{\beta}{p}\right| \sin \Phi_{\beta/p} + L_{p_{L_\delta}}\left|\frac{p}{p}\right| \sin \Phi_{p/p} + \left|\frac{\delta}{p}\right| \sin \Phi_{\delta/p}\right) \end{vmatrix} \begin{vmatrix} L_r \\ L_\delta \end{vmatrix} =$$

$$\begin{vmatrix} \left|\frac{\dot{p}}{p}\right| \cos \Phi_{\dot{p}/p} - \frac{I_{XZ}}{I_X}\left|\frac{\dot{r}}{p}\right| \cos \Phi_{\dot{r}/p} - L_{\beta_o}\left|\frac{\beta}{p}\right| \cos \Phi_{\beta/p} - L_{p_o}\left|\frac{p}{p}\right| \cos \Phi_{p/p} \\ \left|\frac{\dot{p}}{p}\right| \sin \Phi_{\dot{p}/p} - \frac{I_{XZ}}{I_X}\left|\frac{\dot{r}}{p}\right| \sin \Phi_{\dot{r}/p} - L_{\beta_o}\left|\frac{\beta}{p}\right| \sin \Phi_{\beta/p} - L_{p_o}\left|\frac{p}{p}\right| \sin \Phi_{p/p} \end{vmatrix} \tag{9}$$

Matrix (9) can then be solved for L_r and L_δ. Substituting L_r and L_δ into equations (4), the values of L_β and L_p are determined.

All the derivatives of the $\dot{r}$ and a_t equations can be determined in a similar manner.

APPLICATION OF THE TECHNIQUE

The simultaneous time vector derivative identification technique described in this report has been successfully applied to time histories of maneuvers performed with three aircraft.

Figures 1(a) and 1(b) compare flight data from rudder pulse maneuvers with time histories calculated from the explicitly determined time vector derivatives. The maneuver shown in figure 1(a) was performed with the roll SAS off, and the maneuver shown in figure 1(b) was performed with the roll SAS on.

In the analysis of the roll-SAS-off maneuver, the coefficients of the β and p derivatives were determined in terms of the r and δ derivatives, as shown in equations (4).

Analyzing the roll-SAS-on maneuver with the results of the roll-SAS-off maneuver explicitly defines the complete set of stability and control derivatives:

$L_\beta = -5.45\ 1/\sec^2$	$N_\beta = 4.40\ 1/\sec^2$	$Y_\beta = -0.20\ 1/\sec$
$L_p = -0.78\ 1/\sec$	$N_p = 0.02\ 1/\sec$	$Y_p = 0$
$L_r = 0.92\ 1/\sec$	$N_r = -0.12\ 1/\sec$	$Y_r = 0.01$
$L_{\delta_a} = 6.50\ 1/\sec^2$	$N_{\delta_a} = 0.24\ 1/\sec^2$	$Y_{\delta_a} = -0.02\ 1/\sec$

DESIRABLE FEATURES OF THE TECHNIQUE

Some of the desirable features of the simultaneous time vector technique are:

Simultaneous analysis of two maneuvers determines explicitly three stability derivatives and one dependent control derivative in each equation.

It requires only manual analysis of maneuver time histories.

The derivative expressions can be easily programed on a digital computer, eliminating the use of actual vector diagrams.

The technique can be expanded to use more than two maneuvers, making it possible to determine a correspondingly larger number of control derivatives.

No estimates of any derivatives are required.

Zero shift biases of the variables are not required.

A minimum number of variables are required, such as p, r, a_t, and δ for the lateral-directional mode.

Flight Research Center
National Aeronautics and Space Administration
Edwards, Calif., November 10, 1972

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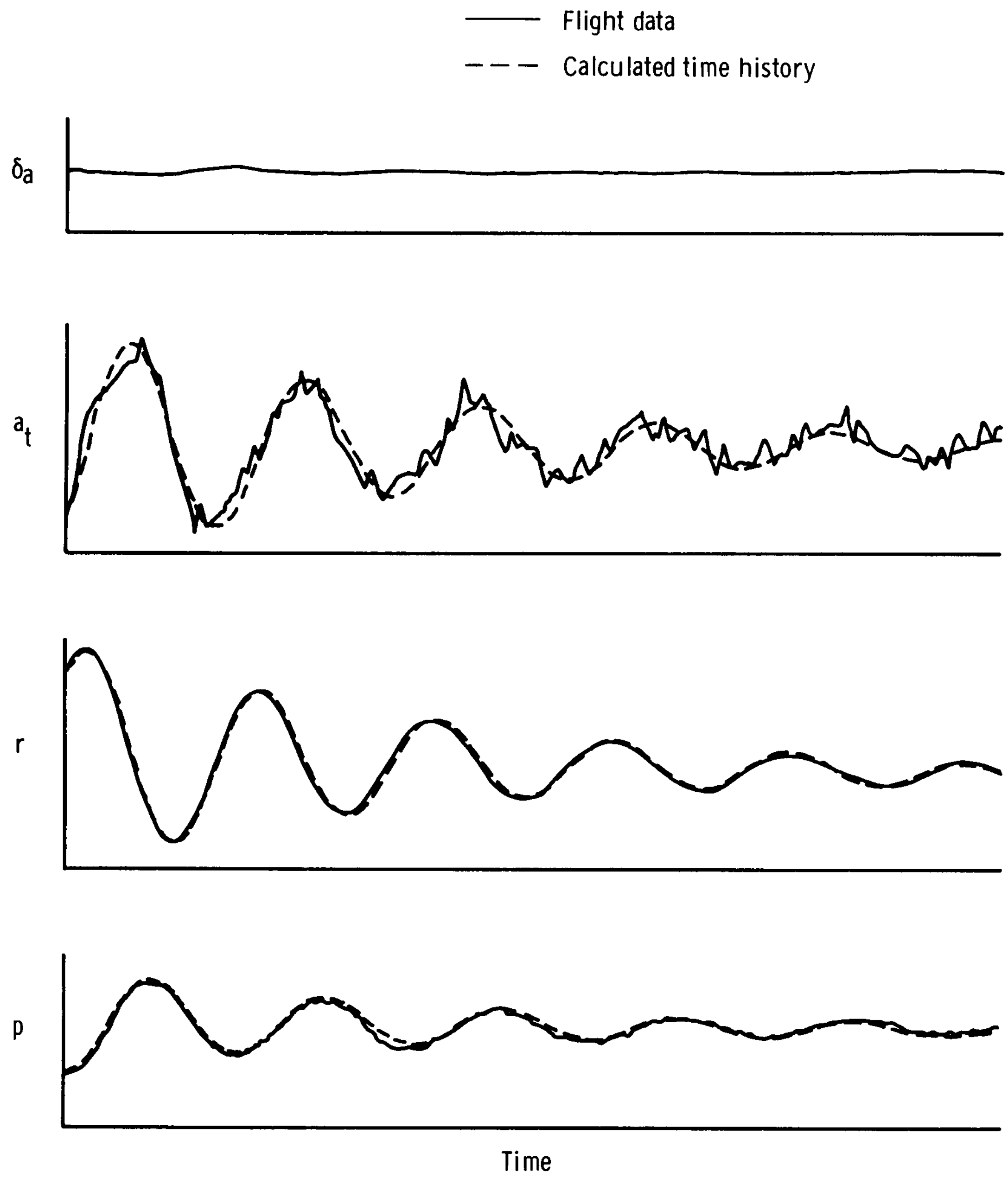
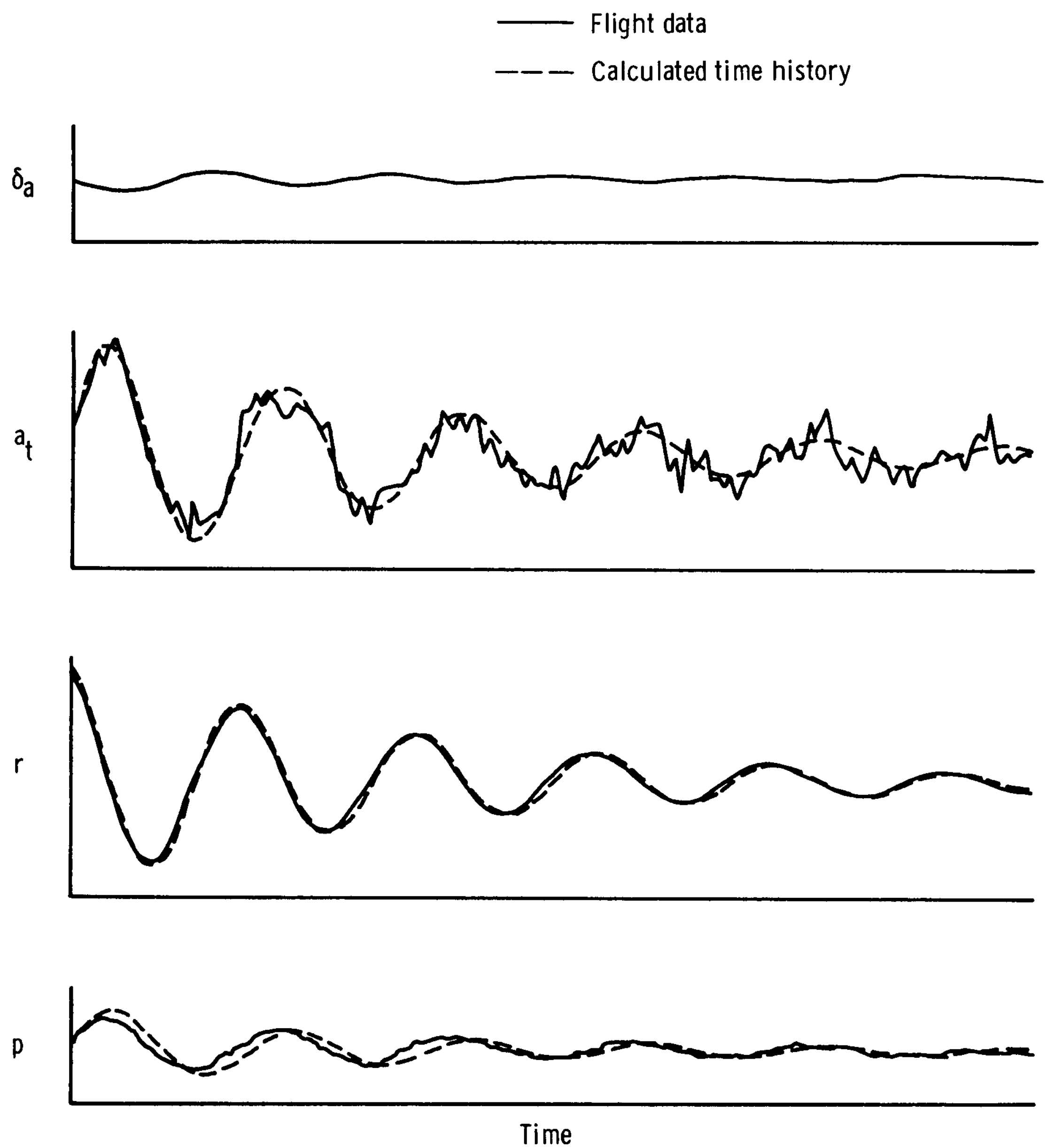


(a) Roll SAS off.

Figure 1. Comparison of lateral-directional flight data with time histories calculated from derivatives determined with the time vector method.



(b) Roll SAS on.

Figure 1. Concluded.